Introduction to AI

Lecture 14
First-Order Logic

Dr. Tamal Ghosh
Department of CSE
Adamas University

Definition of First-Order Logic

•First-order logic is a way of knowledge representation in artificial intelligence. It is an extension of propositional logic.

•FOL is sufficiently expressive to represent the natural language statements concisely.

•First-order logic is also known as **Predicate logic or First-order predicate logic**. Which develops information about the objects in a more easy way and can also express the relationship between those objects.

•First-order logic (like natural language) does not only assume that the world contains facts like propositional logic but also assumes the following things in the world:

- **Objects:** A, B, people, numbers, colors, wars, trees, animals etc.
- **Relations:**
  - **unary relation such as:** red, round, is adjacent,
  - **n-any relation such as:** the sister of, brother of, has color, comes between
- **Function:** Father of, best friend, third inning of, end of etc.

Components of First-Order Logic

- Like natural language, first-order logic also has two main parts:
  - **Syntax**
  - **Semantics**

- Syntax of First-Order Logic:

The syntax of FOL determines which collection of symbols is a logical expression in first-order logic. The basic syntactic elements of first-order logic are symbols. We write statements in short-hand notation in FOL.

- Semantics of First-Order Logic:

Semantics is an interpretation that assigns a denotation to each non-logical symbol. i.e. it defines the relationships among objects.

Basic Elements of First-order logic

The following are the basic elements of FOL syntax:

Constant	1, 2, A, John, Mumbai, cat,....
Variables	x, y, z, a, b,....
Predicates	Brother, Father, >,....
Function	sqrt, LeftLegOf, ...
Connectives	∧, ∨, ¬, ⇒, ⇔
Equality	==
Quantifier	∀, ∃

Atomic sentences

Atomic sentences are the most basic sentences of first-order logic. These sentences are formed from a predicate symbol followed by a parenthesis with a sequence of terms.

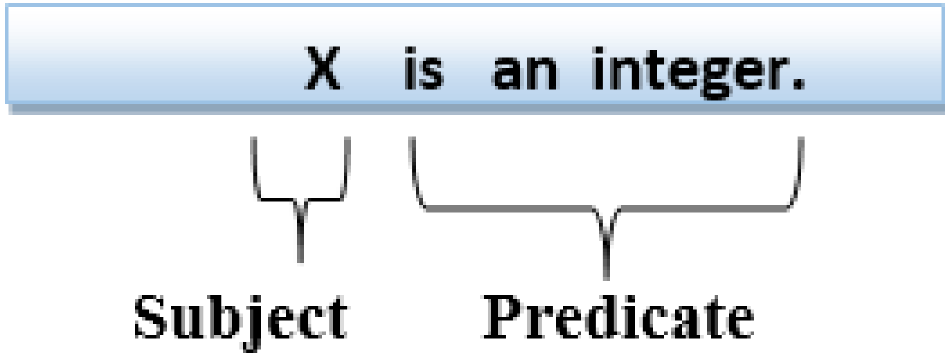
We can represent atomic sentences as **Predicate (term1, term2, ..., term n)**.

Example: Ravi and Ajay are brothers: => Brothers(Ravi, Ajay).

Chinky is a cat: => cat (Chinky).

Complex Sentences

Complex sentences are made by combining atomic sentences using connectives.
First-order logic statements can be divided into two parts:
Subject: Subject is the main part of the statement.
Predicate: A predicate can be defined as a relation, which binds two atoms together in a statement.
Consider the statement: "x is an integer.", it consists of two parts, the first part x is the subject of the statement, and the second part "is an integer," is known as a predicate.



Quantifiers in First-order logic

- A quantifier is a language element that generates quantification, and quantification specifies the quantity of specimen in the universe of discourse.
- These are the symbols that permit to determine or identify the range and scope of the variable in the logical expression. There are two types of quantifiers:
  - **Universal Quantifier, (for all, everyone, everything)**
  - **Existential quantifier, (for some, at least one).**

Universal Quantifier:

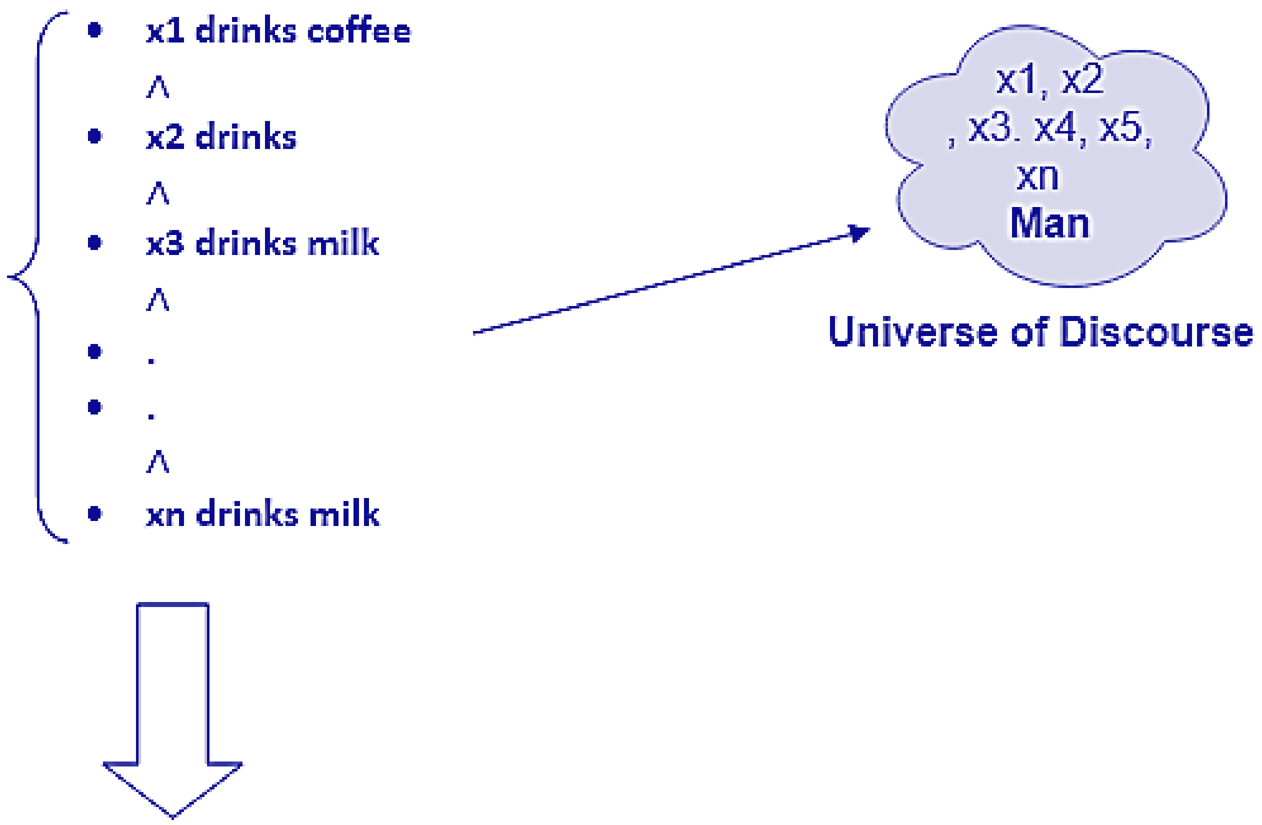
A universal quantifier is a symbol of logical representation, which specifies that the statement within its range is true for everything or every instance of a particular thing.

The Universal quantifier is represented by a symbol $\forall$, which resembles an inverted A.

Types of Inference rules

If x is a variable, then ∀x is read as:
For all x, For each x, For every x.

Example:
All men drink coffee.
Let a variable x which refers to a cat so all x can be represented in UOD as below:



So in shorthand notation, we can write it as :

∀x man(x) → drink (x, y)
It will be read as: There are all x where x is a man who drinks.

Existential Quantifier

- Existential quantifiers are the type of quantifiers, which express that the statement within its scope is true for at least one instance of something.
- It is denoted by the logical operator ∃, which resembles as inverted E. When it is used with a predicate variable then it is called as an existential quantifier.

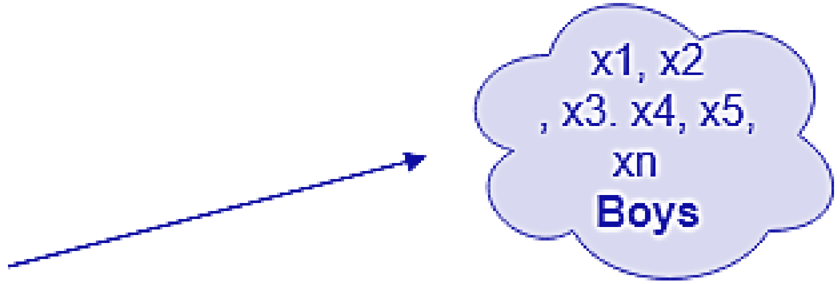
Note: In Existential quantifier we always use AND or Conjunction symbol (∧)

- If x is a variable, then existential quantifier will be ∃x or ∃(x). And it will be read as:
  - **There exists a 'x'**
  - **For some 'x'**
  - **For at least one 'x'**

Example

Some boys are intelligent.

- x1 is intelligent
  V
- x2 is intelligentV
- x3 is intelligent
  V
- .
- .
  V
- xn is intelligent



Universe of Discourse

So in short-hand notation, we can write it as:

∃x: boys(x) ∧ intelligent(x)

It will be read as: There are some x where x is a boy who is intelligent.

Points to remember

- The main connective for universal quantifier **∀** is implication **→**.
- The main connective for existential quantifier **∃** is and **∧**.

Properties of Quantifiers:

- In a universal quantifier, ∀x∀y is similar to ∀y∀x.
- In Existential quantifier, ∃x∃y is similar to ∃y∃x.
- ∃x∀y is not similar to ∀y∃x.

Example

- **1. All birds fly.**
  In this question, the predicate is "**fly(bird)**."
  And since all birds fly so it will be represented as follows.
  ∀x bird(x) →fly(x).
- **2. Every man respects his parent.**
  In this question, the predicate is "**respect(x, y)," where x=man, and y= parent**.
  Since there is every man so will use ∀, and it will be represented as follows:
  ∀x man(x) → respects (x, parent).
- **3. Some boys play cricket.**
  In this question, the predicate is "**play(x, y)**," where x= boys, and y= game. Since there are some boys so we will use **∃, and it will be represented as**:
  ∃x boys(x) → play(x, cricket).

Example (Contd…)

- **4. Not all students like both Mathematics and Science.**
  In this question, the predicate is "**like(x, y)," where x= student, and y= subject**.
  Since there are not all students, so we will use **∀ with negation, so** following representation for this:
  ¬∀ (x) [student(x) → like(x, Mathematics) ∧ like(x, Science)].
- **5. Only one student failed in Mathematics.**
  In this question, the predicate is "**failed(x, y)," where x= student, and y= subject**.
  Since there is only one student who failed in Mathematics, so we will use the following representation for this:
  ∃(x) [student(x) → failed (x, Mathematics) ∧ ∀(y) [¬(x==y) ∧ student(y) → ¬failed (x, Mathematics)].

Free and Bound Variables

- The quantifiers interact with variables within scope. There are two types of variables in First-order logic which are given below:
- **Free Variable:** A variable is said to be a free variable in a formula if it occurs outside the scope of the quantifier.
- **Example: ∀x ∃(y)[P (x, y, z)], where z is a free variable.**
- **Bound Variable:** A variable is said to be a bound variable in a formula if it occurs within the scope of the quantifier.
- **Example: ∀x [A (x) B(y)], here x is the bound variable but y is not.**